On wireless connection between Josephson qubits

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Abstract

By attributing a circulating Josephson current induced diamagnetic moment to a SQUID-type three-level qubit, a wireless connection between such qubits is proposed based only on dipole-dipole interaction between their moments. The estimates of the model parameters suggest quite an optimistic possibility to experimentally realize the suggested coupling scheme.

Josephson qubit is essentially a superconducting ring interrupted by typically two or three Josephson junctions forming the basis of an effective two level quantum system [1, 2, 3, 4, 5, 6, 7, 8]. Most of the recently suggested sign and magnitude tunable couplers between two superconducting flux qubits are based on either direct or indirect inductive coupling mediated by the SQUID [6, 7, 8] (for detailed and up-to-date discussion of different qubit implementations and modern physical coupling schemes, see a comprehensive review article by Wendin and Shumeiko [2]). More precisely [2], in inductive coupling scheme a magnetic flux induced by one qubit threads the loop of another qubit thus changing the effective external flux. This leads to an effective coupling between two-level qubits $H_{int} = \lambda(R)\sigma_1^z\sigma_2^z$ with interaction $\lambda(R)$ dependent on the length of the coupler (and thus qubit separation) R via the mutual inductance $L_{12}(R) \propto R\log R$ as $\lambda(R) \propto L_{12}^{-1}(R)$.

In this report, we propose a wireless coupling between two superconducting Josephson qubits based on dipole-dipole interaction (DDI) D between their diamagnetic moments. Such a dipolar coupler has much in common with the above-mentioned inductive coupling scheme, except that instead of the mutual inductance controlled flux-flux interaction, we have a Zeeman-type proximity-like magnetic interaction of a circulating current I_s^a induced dipole moment m_a of one qubit with an effective magnetic field B_b produced by a dipole moment m_b of the second qubit, and vise versa. Thus, the principal difference of the suggested dipolar coupler from the other coupling schemes is the absence of electric circuit elements (like inductance and/or capacitance) which are known [2] to be the main source of noise and decoherence (dephasing). In this regard, the dipolar coupler is expected to be more "quiet" than its conventional counterparts. Besides, due to the vector nature of the DDI, the sign of such a coupler is defined by the mutual orientation of the qubits while its magnitude varies with the inter-qubit distance as $D(R) \propto 1/R^3$ (see Fig. 1). At the same time, as we shall show, unlike the inductive coupling scheme, the suggested here dipolar coupler requires a three-level qubit configuration for its implementation.

Let us consider a system of two superconducting qubits assuming, for simplicity, that each qubit is a two-contact SQUID with a circulating Josephson current $I_s = I_{c1}\sin\phi_1 + I_{c2}\sin\phi_2$, where $I_{c1,2}$ is the corresponding critical current, and ϕ_1 (ϕ_2) stands for the phase difference through the first (second) contact. In turn, the circulating in each qubit supercurrent I_s creates the corresponding non-zero diamagnetic moment $\mathbf{m} = I_s\mathbf{S}$ with $\mathbf{S}$ being the (oriented) SQUID area. Recall [1] that the quantization condition for the total flux $\Phi =$
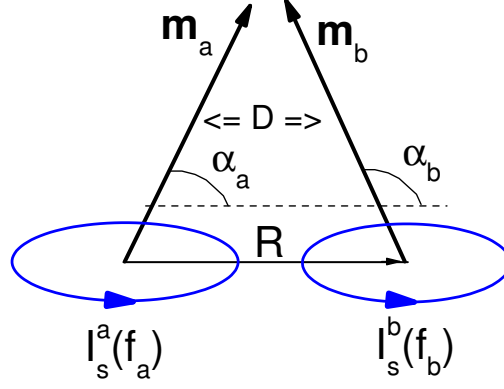
FIG. 1: (Color online) Sketch of a dipolar coupler (with strength D) between currents $I_s^{a,b}$ induced magnetic moments $\mathbf{m}_{a,b}$ of two flux qubits separated by a distance R.

$\mathbf{BS} + LI_s$ (created by the applied magnetic field $\mathbf{B}$ and loop self-inductance L contributions) in each SQUID is given by $\phi_1 - \phi_2 + 2\pi\frac{\Phi}{\Phi_0} = 2\pi n$ with $n = 0, 1, 2, ...$ By introducing a new phase difference θ:

$$\phi_1 = \theta + 2\pi n, \qquad \phi_2 = \theta + \frac{2\pi\Phi}{\Phi_0} \tag{1}$$

we obtain

$$I_s = I_{c1}\sin\theta + I_{c2}\sin(\theta + 2\pi f) \tag{2}$$

and

$$\mathcal{H}_s = -J_1\cos\theta - J_2\cos(\theta + 2\pi f) \tag{3}$$

for the circulating current and tunneling Josephson energy for each SQUID-based qubit, respectively. Here, $f = \Phi/\Phi_0$ and $J = \Phi_0 I_c/2\pi$. In what follows, we neglect the self-inductance of each SQUID, assuming that $LI_s \ll \mathbf{BS}$, and consider $f = \mathbf{BS}/\Phi_0$ as a field-induced frustration parameter. In fact, this condition is rather well met in realistic flux qubits [6, 7, 8] with the so-called degeneracy point $f = 0.5$ and SQUID parameter $\beta_L = 2\pi LI_c/\Phi_0 \simeq 0.1$.

For generality, let us consider two non-identical qubits (a and b) which are assumed to be coupled only via the DDI between their magnetic moments $\mathbf{m}_q = I_s^q S_q \hat{\mathbf{e}}_q$

$$\mathcal{H}_d = \frac{\mu_0}{4\pi R^3}\left[\mathbf{m}_a\mathbf{m}_a - \frac{3(\mathbf{m}_a\mathbf{R})(\mathbf{m}_b\mathbf{R})}{R^2}\right] \tag{4}$$

Here $I_s^q = I_{c1}^q \sin\theta_q + I_{c2}^q \sin(\theta_q + 2\pi f_q)$ is the circulating current in q-th qubit (in what follows, $q = \{a, b\}$), $\mathbf{R}$ is the distance between qubits, and $\hat{\mathbf{e}}_q$ is the unit vector.

Thus, the total Hamiltonian of the two coupled qubits $\mathcal{H}_{tot} = \sum_q \mathcal{H}_s^q + \mathcal{H}_d$ reads

$$\mathcal{H}_{tot} = -\sum_{q=a,b}[J_1^q\cos\theta_q + J_2^q\cos(\theta_q + 2\pi f_q)] + D(f_a, f_b)\sin\theta_a\sin\theta_b \tag{5}$$

where

$$D(f_a, f_b) = D_1\left(1 + j_a\cos 2\pi f_a + j_b\cos 2\pi f_b + j_a j_b\cos 2\pi f_a\cos 2\pi f_b\right) \tag{6}$$

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with

$$D_1 = D_0 \left(2 \cos \alpha_a \cos \alpha_b - \sin \alpha_a \sin \alpha_b\right) \tag{7}$$

Here $D_0 = (J_1^a/2\pi)(R_0/R)^3$ with $R_0 = \sqrt[3]{4\pi^2\mu_0 J_1^b S_a S_b/\Phi_0^2}$ being a characteristic distance between qubits, $j_q \equiv J_2^q/J_1^q$, and $\alpha_{a,b}$ are the angles of $\mathbf{m}_{a,b}$ relative to the distance $\mathbf{R}$ between qubits. A sketch of the proposed dipolar coupler is shown in Fig. 1. Notice that, due to its vector character, the DDI naturally provides a sign-dependent coupling between the qubits which is quite similar to conventional SQUID inductance mediated coupling [6, 7, 8]. It is also interesting to mention that, in view of Eq.(5), DDI automatically results in a non-conventional current-phase relation [9] $I_d(\theta_a) = \partial\mathcal{H}_d/\partial\theta_a \propto D \cos\theta_a \sin\theta_b$ (with a sign-changeable amplitude D) usually observed in SFS structures and attributed to the formation of π-type contacts [10, 11].

A careful analysis of the structure of the total Hamiltonian $\mathcal{H}_{tot}$ reveals that the inter-qubit dipole coupling D introduces the transitions (mixing) between more distant states, namely $|0>$ and $|2>$ suggesting thus that implementation of DDI requires a three-level qubit configuration [12, 13, 14] (instead of its more traditional two-level counterpart [1, 2, 3, 4, 5, 6, 7, 8]). The resulting three-level system can be readily cast into the following form of the qubit Hamiltonian

$$\mathcal{H}_Q = -\sum_{q=a,b} \left(\epsilon_q M_q^z + \Delta_q M_q^x\right) + D \left(M_a^z M_b^z + 2M_a^y M_b^y\right) \tag{8}$$

where $\epsilon_q = (J_1^q + J_2^q)\sin(\delta_q) \simeq (J_1^q + J_2^q)\delta_q$ and $\Delta_q = (J_1^q + J_2^q)\cos(\delta_q) \simeq (J_1^q + J_2^q)$ are the energy bias and tunneling splitting for the q-th qubit. Here $\delta_q \equiv 2\pi(f_q - \frac{1}{2}) \ll 1$.

$$M_q^x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, M_q^y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, M_q^z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

are the spin-1 analog of the Pauli matrices σ_q^α.

It is easy to verify that in the absence of coupling (when $D = 0$) each individual qubit has three energy levels: $E_1^0 = 0$ and $E_{2,3}^0 = \pm\sqrt{\epsilon_q^2 + \Delta_q^2} \simeq \pm\Delta_q(1 + \frac{1}{2}\delta_i^2)$, in accordance with the M_q^z structure.

At the same time, the energy levels of the coupled qubits can be found by solving the eigenvalue problem $\mathcal{H}_Q\Psi = E\Psi$. As a result, we obtain the following cubic equation on the
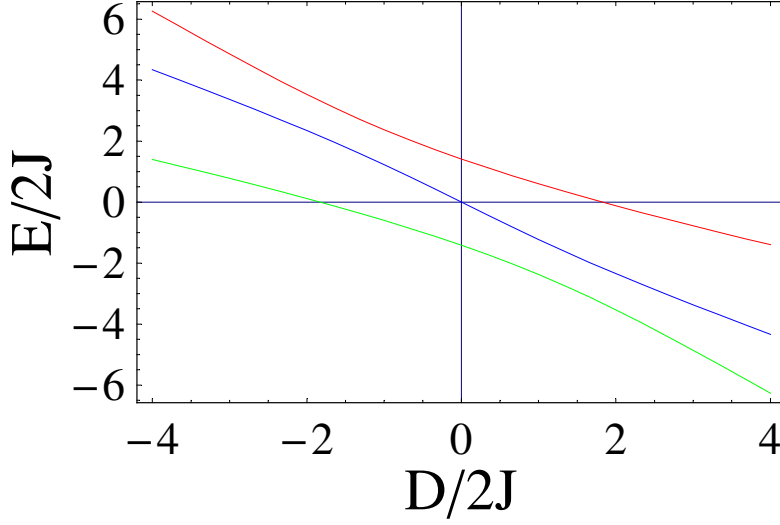
FIG. 2: (Color online) The dependence of the normalized energy levels E_1, E_2 and E_3 (from bottom to top) on dipole-dipole coupling $u(0,0) = D/2J$ for $f_a = f_b = 1$.

energy spectrum E of the problem (as a function of two controlling parameters, D and f_q)

$$2\Gamma^3 - \left(4\Delta^2 + \frac{1}{2}D^2\right)\Gamma - D\Delta^2 = 0 \tag{9}$$

where $\Gamma = D - E$ and $\Delta = \sum_q \Delta_q$. It can be directly verified that the above cubic equation has the following three independent solutions $E_{1,2,3}$ (corresponding to the three-level qubit configuration), namely

$$E_1 = \Delta\left[u - (a_+ + a_-)\right] \tag{10}$$
$$E_{2,3} = \Delta\left[u + \frac{(a_+ + a_-)}{2} \pm i\sqrt{3}\frac{(a_+ - a_-)}{2}\right]$$

where

$$a_\pm = \sqrt[3]{\frac{u}{4} \pm \frac{1}{4}\sqrt{u^2 - \frac{1}{4}\left(\frac{8}{3} + \frac{u^2}{3}\right)^3}} \tag{11}$$

and $u(f_a, f_b) = D(f_a, f_b)/\Delta$.

Without losing generality, in what follows we assume that $J_1^q = J_2^q \equiv J$ (which means that $j_q = J_2^q/J_1^q = 1$ and $\Delta = 2J$) and that $f_a = f_b \equiv f$. Fig. 2 shows the dependence of the normalized energy levels $E/2J$ on the dipole-dipole coupling $u(0,0) = D/2J$ for $f_a = f_b = 1$. Notice that $u(0,0)$ can assume negative values due to vector nature of the dipole interaction D. In turn, Fig. 3 depicts the evolution of the coupled three-level qubits
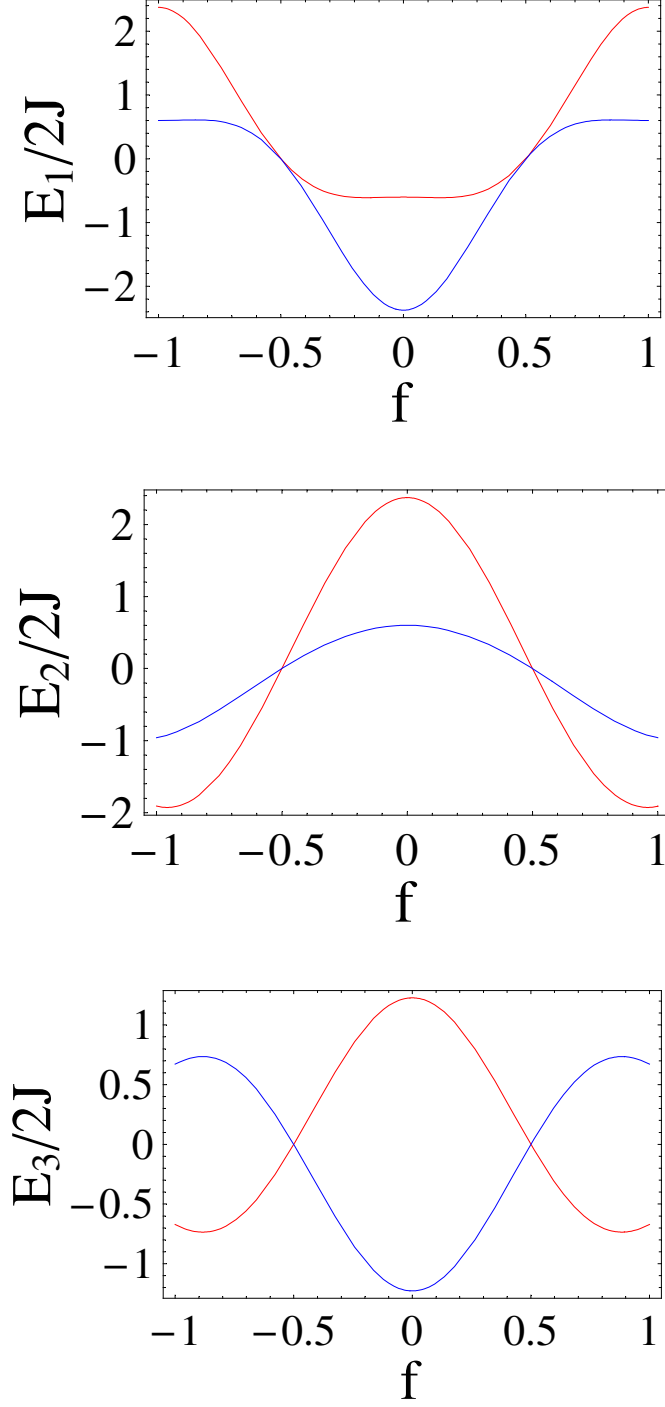
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FIG. 3: (Color online) The dependence of the normalized energy levels $E_{1,2,3}/2J$ on frustration parameter $f_a = f_b \equiv f$ for two values of the dipole-dipole coupling: $u(0,0) = 1$ (bottom) and $u(0,0) = -1$ (top).
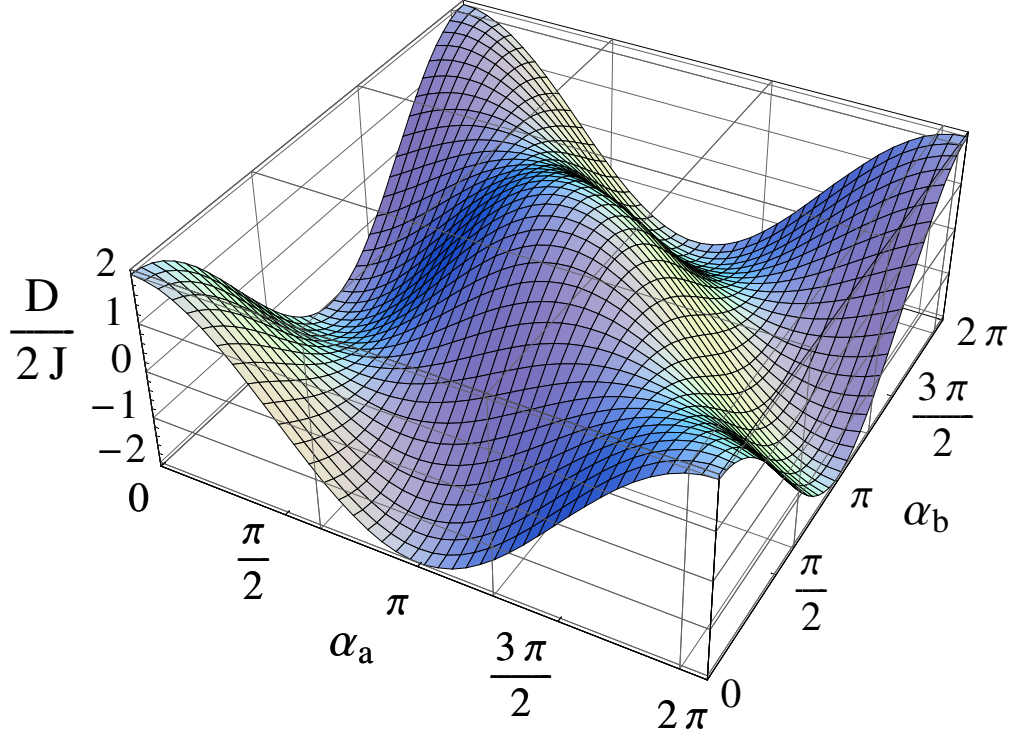
FIG. 4: (Color online) The dependence of the dipolar coupler strength on angles between qubits shown in Fig. 1

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with applied magnetic field (frustration parameter f) for two different qubits orientations (given by $u(0,0) = 1$ and $u(0,0) = -1$, respectively). As would be expected for flux qubits [1, 2], the degeneracy point is situated near $f = 0.5$. The angular dependence of the DDI amplitude D on $\alpha_{a,b}$ is shown in Fig. 4. Notice that, like in inductive based coupling scheme [2, 6, 7, 8], the dipolar coupler may change its sign from positive (when two moments are parallel to each other) to negative (for the anti-parallel configuration) or even disappear (when two moments are perpendicular to each other). In turn, Fig. 5 depicts variation of $D_1(\alpha_q, R)$ as a function of the normalized distance R/R_0 between qubits for three values of α (assuming the parallel orientation with $\alpha_a = \alpha_b = \alpha$, see Fig. 1). Finally, let us estimate the main model parameters based on the available experimental data on long-range couplers. Using [7] $S = 50\mu m \times 50\mu m$ and $I_c = 0.5\mu A$ for the area of a single qubit and SQUID critical current, we obtain $R_0 = \sqrt[3]{2\pi\mu_0 I_c S^2/\Phi_0} \simeq 100\mu m$ for DDI characteristic separation
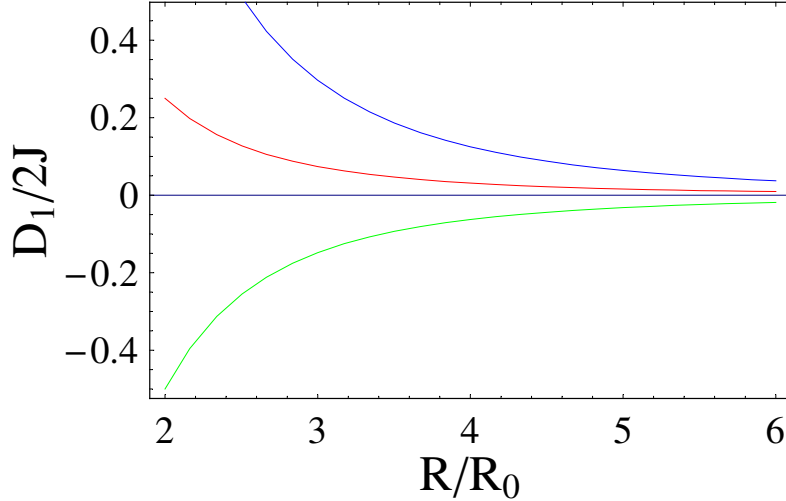
FIG. 5: (Color online) The dependence of $D_1(\alpha_q, R)$ on the distance R between qubits for three values of α (from top to bottom: $\alpha = \frac{\pi}{2}$, $\frac{\pi}{3}$, and $\frac{\pi}{4}$).

which results in the following estimate for DDI mediated coupler frequency (see Fig. 5) $\Omega = D/h = (J/2\pi h)(R_0/R)^3 \simeq 1GHz$ for the inter-qubit distance of $R = 400\mu m \simeq 4R_0$. This value remarkably correlates with the frequencies achieved in the mutual inductance mediated couplers [2, 6, 7, 8], suggesting quite an optimistic possibility to experimentally realize the proposed here dipolar coupling scheme.

In summary, a theoretical possibility of wireless connection between Josephson qubits (based on dipole-dipole interaction between their induced magnetic moments) was proposed and its experimental realization was briefly discussed.

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